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On the question of possible experimental observation of Anderson localization of the neutron

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Abstract. A possible experiment is discussed, for the observation of Anderson localization of the neutron. The localized state may be formed in the process of inelastic downscattering of thermal or cold neutrons in a highly disordered substance with low neutron capture and upscattering cross sections. The lifetime of trapped (localized) neutrons in the sample is measured by counting the upscattered neutrons with a neutron counter surrounding the sample. Estimations of experimental parameters relevant to such an experiment are given.

PACS. 66.90.+r Other topics in nonelectronic transport properties of condensed matter – 28.20.Gd Neutron transport; diffusion and moderation – 78.90.+t Other topics in optical properties, condensed matter spectroscopy and other interactions of particles and radiation with condensed matter

1 Introduction

The phenomenon of the localization of quantum states in random media was first described years ago by Anderson in his famous paper [1]. He considered the quantum mechanical diffusion of electrons in a disordered solid and found that electron diffusion is absent in certain random lattices. This discovery has its widest and most striking experimental manifestations in the transport properties of electron in condensed matter systems [2]. It was recognized that a number of phenomena may be understood in terms of the localization of electronic wave functions in space.

The dominant mechanism for the localization of states in a random medium is coherent multiple scattering and quantum interference. Therefore, the localization effect should also be of importance in other wave phenomena. This was demonstrated theoretically [3] by considering classical wave equations, it was then noted by several authors [4–7] that localization should occur for classical waves of different natures (electromagnetic, acoustic) propagating in a disordered medium. Up to now only the precursive effects of localization, in the form of the enhanced backscattering of visible light by aqueous suspensions of polystyrene spheres have been observed [8,9]. This regime of weak localization is the best one understood theoretically because perturbation techniques are applicable. In experiments on the propagation of visible [10, 11] and microwave range elecromagnetic radiation [12,13] evidence of nonclassical diffusion was found. However, as was shown in [14] both theoretically and experimentally, some of these results can be explained without recourse to strong localization when resonance scattering is properly taken into account, which may lead to very low transport velocities.

During the last decade the theoretical and experimental situation regarding the localization of classical waves (light, sound) was extensively discussed, but no final prediction was made for the problem of wave transport in the localization regime. The complete theory of localization is still lacking. It has been solved for the one-dimensional case and significant progress has been achieved in two-dimensional localization theory. For three-dimensional case, the situation is still undetermined. For example no definite way has been proposed to calculate the density of states or the value of the localization length for an arbitrary disordered medium.

Traditionally, during wave propagation in disordered media, scattering takes place on scales much longer than the wavelength. In this classical diffusion limit, in which $kl \gg 1$, where k is the wave vector and l is the elastic mean-free path, the phases of scattered waves are uncorrelated and propagation may be described in terms of diffusion of the particle density. The diffusion coefficient in this case is $D = vl/3(1 - \langle\cos\theta\rangle)$, where $l/(1 - \cos\theta)$ is the transport free path length. Localization of waves may occur when the scale of coherent multiple scattering is reduced to the wavelength: $kl \sim 1$. In this case (the strong scattering regime) extended correlations in scattered waves lead to destructive interference, which reduces the average transport rate, and the diffusion coefficient is determined by the scale $(l - l_c)$ instead of l, where according to [15] $l_c \simeq k^{-1}$. When l reaches a critical value l_c, the waves are localized in the sense of Anderson [1]

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that $D(L) \to 0$ as $L \to \infty$, where L is the sample size. Experimental evidence of localization manifests itself in the special character of the transmission T of an inhomogeneous layer of thickness W in the presence of incipient localization, which changes from $T = l/W$, (classical diffusion) to $(l/W)^2$ (critical regime), and then to $\left((l_c - l)^2/W^2\right) \exp\left(-W/L_{loc}\right)$.

2 Proposed method

Meshcherov [16] proposed experiments for transmitting neutrons through inhomogeneous media for the purpose of demonstrating the localization phenomenon for neutrons. The neutron energy convenient for observing the localization is in the ultracold range ($E \simeq 10^{-7} - 10^{-6}$ eV), for which the refraction index $n = (1 - k_0^2/k^2)^{1/2}$ maximally differs from unity and scattering from inhomogeneities of the media is the strongest. The localization state is not reached in this type of experiments as the wave is almost totally reflected from the region of localization and penetrates through the sample only due to the exponential tail of the localized wave function $\psi \sim \exp(-x/L_{loc})$.

Transmission experiments may be extremely difficult to perform and may be not decisive, especially in case when the localization length is not very great. For example, if $L_{loc} \leq (10^2 - 10^3)k^{-1} \simeq (1 - 10)$ μm, at a sample thickness of only $(10 - 100)$ μm, the transmitted intensity will be e^{-10} of the primary beam flux. An additional serious problem is to have a very thin homogeneous (without holes) sample. On the other hand, for very large localization lengths, of the order $(10^5 - 10^6)k^{-1} \simeq (0.1 - 1)$ cm, the exponential dependence of the transmission on the sample thickness may be determined by inelastic UCN losses.

In this article we would like to propose another type of experiment, where it may be possible to form the localized state of a neutron directly. This may be achieved through inelastic downscattering of the beam of slow neutrons in disordered media. According to the sense of localization of the particle, its probability density exponentially decays outside a certain "region of localization". Localized particles have exponentially small chances of running away from a random system. Any particle outside the localized energy band has an exponentially small probability of getting inside a random system. Neutrons localized in this way may be captured or inelastically upscattered to the thermal (or cold) energy range with a time constant dependent on the corresponding cross sections: $\tau = (\sum_i n_i \sigma_i v)^{-1}$, where n_i is the mean number per cm^3 of atoms with the cross section σ_i, v is the neutron velocity, and $\sigma = \sigma_{inel} + \sigma_a$ is the sum of upscattering and nuclear capture cross sections. The most convenient substances for realizing such experiments are strong coherent neutron scatterers with low capture and upscattering cross sections (at low temperatures). The calculated time constants are given in Table 1 for several substances with their effective Debye temperatures T_D [17] and corresponding sample temperatures T_s. In these calculations, the density of the disordered medium was taken to be one tenth of the normal density, the temperatures of the samples were chosen such that the upscattering cross section is approximately equal to the capture cross section, the former being calculated in the incoherent one-phonon approximation [18] with the experimental frequency spectrum [19] for beryllium and graphite, and Debye frequency model for BeO and deuterium. For D_2O and CO_2, the "optimal" temperatures were not calculated in this work. It was assumed that the frequency spectrum and Debye temperatures do not change significantly in the disordered form. The sample temperatures may be taken only for orientation in view of mentioned approximations.

Table 1. UCN mean life times in some substances at one tenth of normal density.

Substance	Be	BeO	C (graphite)	D_2	D_2O	CO_2
T_D(K)	-	1200	-	114	-	-
T_s(K)	98	130	72	5	-	-
τ(ms)	25	38	58	850	550	260
τ_{diff}(ms)	19	25	33	69	69	58

After irradiation of the sample by a beam of slow neutrons, the beam is closed and the time dependence of the upscattered neutron intensity is measured with neutron counters located around the sample. In order to distinguish true localization from classical diffusion, the sample must be performed in such a manner that diffusing neutrons are captured significantly sooner in the case of diffusion than in the presence of localization, when the particle does not leave the microscopic site the size of the localization length. Two sample arrangements are possible, with one- and two-dimensional arrangements of the neutron absorbers inside the sample. In the first arrangement thin plane foils of proper absorbing material are placed between thin layers of disordered media. In the second arrangement, the disordered media has the form of long thin cylinders inside the absorbing casings. The Fourier method calculations of the nonstationary diffusion of particles, homogeneously distributed over the volume, to the absorbing boundaries, give the following characteristic particle density decay time constants: $\tau_1 = [v(1/l_a + \pi^2 l_s/3d^2)]^{-1}$ for the first case and $\tau_2 = [v(1/l_a + 4\alpha^2 l_s/3d^2)]^{-1}$ for the second. Here v is the particle velocity, l_a is the mean capture length for the particle, d is the thickness of the disordered layer in the first case and the cylinder diameter in the second one, and $\alpha = 2.405$ is the first root of the Bessel function J_0. In the fourth row of Table 1, the calculated UCN lifetimes are shown, accounting for neutron diffusion to the absorbing boundaries for one-dimensional case: $v = 10$ m/s, $d = 0.05$ cm, $l_s = 10^{-5}$ cm.

The density of trapped (localized) neutrons may be estimated using the following expression for the rate $P(E_{UCN})$ of UCN production into the unit energy

Table 2. The rate of UCN production in some substances at one tenth of normal density in the energy interval 0.1 μeV at $E_{UCN} = 0.525$ μeV.

Substance	Be	C (graphite)	D_2	BeO
T_s(K)	98	72	5	130
E_n(meV)	25	25	4	30
$P(\mathrm{s}\cdot\mathrm{cm}^3\cdot 0.1\ \mu\mathrm{eV})^{-1}$	0.32	0.07	13	0.11

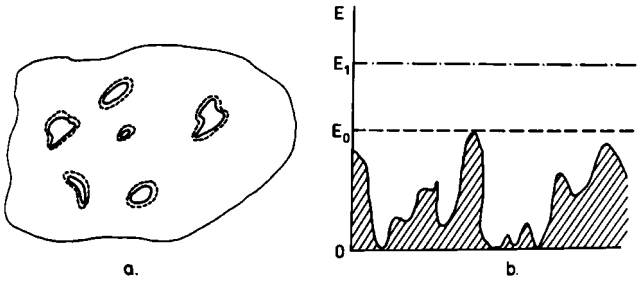

Fig. 1. (a) The UCN with energies below the boundary energy E_0 must be trapped according [20] into micropores by tunneling from thin layers (thickness $\sim \lambda \sim 100$ Å) surrounding the pores. (b) For highly disordered media, multiple coherent scattering at the potential fluctuations may lead to a superposition of destructively interfering waves in such a way that the neutron may become localized in the energy interval $[E_0, E_1]$.

interval:

$$P(E_{UCN}) = n \int \Phi(E)\sigma(T, E \to E_{UCN})\rho(E^{loc}_{UCN})dE, \tag{1}$$

where $\Phi(E)$ is the primary neutron flux density, $\sigma(T, E \to E_{UCN})$ is the cross section for the downscattering of neutrons with energy E in the primary beam on the nuclei of the sample into the unit UCN energy interval, n is the mean number per cm^3 of the sample, and $\rho(E^{loc}_{UCN})$ is the density of localized UCN states.

Table 2 gives the values of $P(E_{UCN})$ for several substances at corresponding "optimal" temperatures T_s. The final energy is 0.525 μeV, ($v = 10$ m/s), the UCN energy interval was taken as 0.1 μeV, the density of samples was taken as one tenth of the normal density, and the primary neutron flux had the Maxwellian form $\Phi(E) = \phi_0 E \exp(-E/E_n)/E_n^2$, with $\phi_0 = 10^{10}$ n/cm^2/s and E_n given in Table 2. The localized UCN density of states is not known, so we took it to be equal to the density of states for a free neutron.

The present proposal is related to the paper published years ago [20] that proposed trapping neutrons in samples with micropores. The proposal was made in connection with experiments [21] (later proven to be incorrect [22]) for the observation of very long (tens of sec) lifetimes of slow neutrons in LiF crystals. The difference between these two variants may be seen in Figure 1. The UCN with energies below the boundary energy E_0 must be trapped according [20] into micropores by tunneling from the thin layer (thickness $\sim \lambda \sim 100$ Å), surrounding the pore. Classically, in not strongly disordered media particles with energy $E > E_0$ are not trapped by the medium. For strongly disordered media, multiple coherent scattering at the potential fluctuations may lead to a superposition of destructively interfering waves in such a way that neutrons may become localized in the energy interval $[E_0, E_1]$. As was mentioned, the theory of localization in its modern state is not able to predict the value E_1 or the density of localized states in this energy band.

The crucial question is the density of localized states in equation (1) which determines the possible density of trapped neutrons. In the case where the transmission experiments [16] are successful, the proposed method of observation of Anderson localization may answer this question.

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